

Energy Management and Optimal Operation of Light Rail Trains on Jerusalem's Proposed Green Line

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ABSTRACT This work presents an integrated optimization framework for the operation of urban light rail systems, incorporating detailed traction dynamics, energy exchanges, battery behavior, and realistic infrastructure constraints into a unified mathematical model. Notably, the model accounts for the energy recovered through regenerative braking, significantly reducing operating costs. The formulation results in a non-linear problem solved efficiently with commercial solvers, and its flexible structure allows adaptation to different railway technologies, grid configurations, and operating strategies. The model was applied to the Jerusalem Light Rail Green Line using detailed topographic data and over 2,000 track segments covering a 32.7 km round trip, enabling full-cycle energy and operational assessment under realistic service conditions. Sixteen realistic operational scenarios were simulated, featuring cost or time minimization, onboard energy storage system (ESS) availability, regenerative braking, and different maximum speeds. Additionally, sensitivity analyses were carried out to quantify the influence of key parameters on operational performance and costs. Results show that time minimization produces small travel time reductions, while considerably increasing energy cost. In contrast, cost minimization yields a more balanced and sustainable trade-off between performance and efficiency. The best configuration corresponds to cost minimization with both regenerative braking and ESS. Detailed analysis reveals how the ESS smooths power fluctuations, enhances energy recovery, and limits purchases from the grid. Finally, the model was extended to full-day operation under realistic timetables with 5-minute frequencies; in this configuration, daily operating costs reach €8,273.52, with €270.14 saved thanks to regenerative braking. These results demonstrate the practical applicability of the framework and offer actionable insights and practical guidance for railway operators.

INDEX TERMS Energy management, energy storage, light rail train, optimization, regenerative braking, simulation.

I. INTRODUCTION

The urgent global imperative to reduce greenhouse gas emissions, combat urban air pollution, and decrease reliance on fossil fuels has led governments worldwide to invest heavily in clean and efficient public transport systems. Urban rail, encompassing light rail and metro, has emerged as the backbone of sustainable urban mobility, offering high energy efficiency, enhanced safety, and the capacity to significantly

reduce private vehicle use in dense metropolitan areas [1], [2].

In Israel, urban rail development symbolizes a strategic shift from car-centric infrastructure toward integrated transit networks. The Jerusalem Light Rail Red Line, from Neve Yaakov to Hadassah Ein Kerem, became fully operational in February 2025 on a 22.5 km corridor electrified at 750 V DC, serving more than 130000 daily passengers. The Green

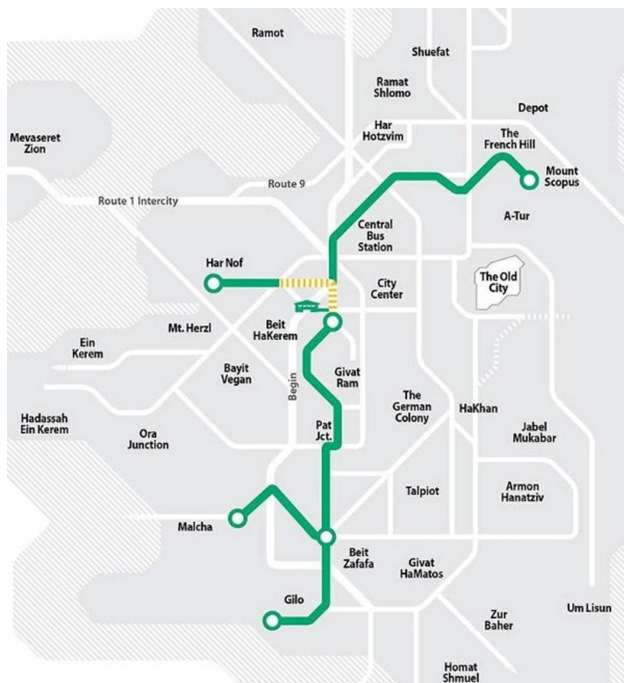


FIGURE 1. Green line in J-Net. Source: CAF-shapir.

Line, currently under construction, with completion expected in 2027, will span 19.6 km with 34 stations between Hebrew University and Gilo, sharing sections with the Red Line and linking to Mount Scopus. In August 2019, the TransJerusalem J NET consortium, led by CAF and Shapir, secured a €1.8 billion Public-Private Partnership contract to design, build, and operate the Green Line, see Fig. 1. With anticipated ridership of around 200000 daily passengers, the Green Line is poised to enhance urban connectivity, increase transit capacity, and diminish private car dependency [3], [4], [5]. This line will be served by CAF Urbos-100 trams that will run on 750 V DC overhead lines, combining hydraulic/magnetic braking with regenerative capabilities [6].

Modern electric rail systems, especially those operating on DC urban networks, enable substantial energy recovery via regenerative braking. From a control-system perspective, regenerative braking in light rail vehicles is implemented through a hierarchical architecture in which the Train Control and Monitoring System (TCMS) provides supervisory commands, while the execution of traction and braking actions is handled locally by the traction and braking subsystems. Regenerative braking is realized by operating the traction motors in generator mode during deceleration, with the resulting electrical energy injected into the DC traction network, subject to system constraints such as DC voltage limits, converter current limits, adhesion conditions, and safety interlocks. The control logic is generally based on industrial rule-based strategies and constraint enforcement with automatic blending between electrical and mechanical braking to ensure braking performance and safety, rather than real-time optimization algorithms or strictly pre-established optimal

operating schedules. The physical hardware responsible for this functionality is located onboard the train, mainly within the traction converters and braking control units, while energy reuse—either by other accelerating trains or through the network—is managed implicitly by the electrical characteristics of the DC system. Finally, low-level traction and braking control is beyond the scope of this work, which instead focuses on system-level energy management and the optimization of regenerative energy utilization at the network level; therefore, some simplifications are introduced to model the energy exchange.

When traction motors act as generators during braking, they return electricity to the network or dedicated storage systems [7]. Empirical studies consistently show that regenerative braking can reduce total energy consumption by 10–35% when well-integrated with infrastructure and synchronized train schedules [1]. The net benefit heavily depends on headway intervals, substation distance, substation reversibility, supply type (catenary vs. third rail), and the ability of neighboring trains or storage mediums to absorb regenerated energy. Extensive literature examines these variables, including substation location [8], real-time traction power control [6], timetable optimization for regenerative braking energy maximization [9], [10], [11] and simulation-supported speed-profile and storage integration analyses [12], [13], [14], [15].

Mathematical modeling and simulation-driven studies have played a crucial role in designing and validating railway infrastructure and regenerative systems in particular [1], [16], [17], [18], [19]. These works have explored both static and dynamic modeling approaches to evaluate system performance under varying operational conditions, including train density, scheduling, and energy flow management. Linear programming formulations have been applied to optimize traction energy consumption under simplified operational assumptions [17]. A 2024 study showed that optimizing metro timetables enhances the efficiency of regenerative braking energy utilization by improving synchronization between trains [18]. The authors of [9] and [20] analyzed the performance of the regenerative braking in real implementations, demonstrating noticeable improvement in overall energy efficiency. The authors of [21] and [22] propose optimization problems for the sizing of onboard storage featuring supercapacitors. The work by Domínguez et al. [23] provides a comprehensive overview of system-level recovery strategies, evaluating both economic and technical feasibility. Additionally, Chen et al. [24] proposed a two-stage optimization model that coordinates train operations with electric vehicle (EV) charging infrastructure in suburban railways, demonstrating that smart integration with the local grid can further enhance overall energy efficiency and recovery potential. Similarly, van der Horst et al. [25] explored DC traction grids as multi-purpose energy platforms, proposing methods to integrate EV chargers directly into the catenary network to improve overall system flexibility and energy reuse.

Recent studies have highlighted the role of wayside energy recovery systems in maximizing the reuse of braking energy

within DC urban railway networks, especially when integrated with reversible substations and optimized train headways [20], [23], [26]. The work by Meishner and Sauer [27] first outlined the operational principles and performance of wayside storage systems in DC traction grids, providing a practical benchmark for subsequent optimization studies. In parallel, onboard energy management technologies have advanced substantially, extending regenerative principles beyond fixed infrastructure. Peng et al. [28] developed a co-optimization framework for fuel cell hybrid trains that integrates total running time, timetables, driving strategies, and energy management, while Peng et al. [29] introduced a scalable, adaptive rule-based energy management strategy for hybrid railway vehicles.

While several prior studies have examined the potential of regenerative braking to enhance the energy efficiency of urban rail systems, many have relied on generalized simulations or partial system data. For instance, in [18], the authors demonstrated that timetable optimization in a metro system can improve the utilization of regenerative energy; however, their model did not incorporate detailed track profiles or analyze train-to-train interactions in opposite directions. Other studies have focused primarily on aggregate consumption data, without considering line topology or substation distribution, limiting the accuracy of energy flow assessment along the network. The authors of [23] review a range of energy storage technologies, such as supercapacitors and batteries, in urban DC rail networks, but concentrated mainly on technological architecture rather than structural or operational impacts of track infrastructure. The authors of [30] have addressed the optimal placement of energy storage systems in electrified railways, considering several income streams for the operator. In [31], the authors propose a two-layer optimization approach to manage the use of stationary energy storage systems coupled with railway operation, reducing costs and increasing efficiency. Similarly, Alnuman et al. [32] developed a detailed DC railway simulation model to determine the optimal placement of wayside energy storage systems for improving energy efficiency and power management; however, the study did not include a detailed simulation of the network, as addressed in the present work.

The present research is grounded in a high-resolution dataset composed of 2154 highly granular track segments, each mostly under 20 meters in length (with a median of ~ 10 meters). Each entry includes precise geometric and operational parameters such as slope, curvature, station location, operational mode (acceleration, braking, coasting), and energy flow. Moreover, over 60 segments in the right-hand track and 70 in the left-hand track are shorter than one meter, highlighting the exceptional granularity of the dataset, far beyond what is typically reported in the literature. This level of detail enables accurate modeling of grade transitions, braking zones, and regenerative energy timing windows.

As previously mentioned, recent studies on railway energy efficiency have demonstrated that timetable optimization [9], [11], [13], [18] can be a useful tool in order to improve train operation; however, these approaches are limited as they

mainly address scheduling decisions and do not explicitly incorporate detailed train motion dynamics, onboard energy storage sizing and operation, or infrastructure-constrained power exchange. The present work goes beyond timetable-oriented optimization by proposing an integrated operational framework that simultaneously captures train dynamics, regenerative braking, onboard energy storage behavior, and electrical infrastructure constraints. Unlike timetable-based studies, the proposed model explicitly evaluates both time-minimizing and cost-minimizing operation strategies,

Additionally, the model explicitly incorporates the location of rectifier substations and computes voltage drops across the infrastructure, both of which are critical to assessing the network's real-time capacity to absorb and redistribute recovered energy. In our work, using advanced optimization tools, several scenarios are modeled that simulate a loop-based rail configuration, where trains travel simultaneously in opposite directions along physically distinct tracks with separate geometries. Accordingly, this study is able to quantify the energy recovery potential in a bidirectional DC light rail corridor using high-resolution topographic data and an integrated MATLAB–GAMS simulation framework.

Finally, in our study, the framework is extended from single-trip optimization to full-day operation under realistic service frequencies, allowing the quantification of daily operating costs and the benefits of regenerative energy sharing between trains. Moreover, the proposed framework is explicitly constrained by rolling-stock performance limits derived from manufacturer-provided technical documentation, including traction and braking current and voltage limits, maximum tractive effort, acceleration bounds, auxiliary power demand, and motion resistance parameters. This ensures that the feasible operating space explored by the optimization reflects real vehicle capabilities rather than assumed or generic values commonly adopted in prior studies. This operationally grounded and infrastructure-aware perspective clearly distinguishes the proposed approach from existing timetable-based energy optimization studies.

In summary, this work addresses regenerative energy utilization in DC urban rail systems through three main contributions: (i) a system-level optimization framework that represents the interaction between train operation and the electrical network under realistic constraints; (ii) the use of high-resolution infrastructure data to model bidirectional operation and energy exchange along the line; and (iii) an extension of the analysis to full-day service conditions, enabling the assessment of energy recovery as a function of service frequency. This last aspect is particularly relevant for system operators, as it provides quantitative support for frequency planning decisions while accounting for infrastructure and vehicle limitations.

Section II presents the mathematical modeling approach; Section III presents the case study and the results obtained and Section IV discusses results and implications and draws some conclusions.

II. NOTATION

The following symbols have been used through the document and are presented here for reference. *A. Indexes*
 r index for track sections.

B. Parameters

CBU	Acquisition cost of batteries [€/MWh].
B_L	Expected useful life of the batteries [years].
CRM	Maximum value of the C_rate parameter used to operate the battery; it establishes a relation between energy capacity and power exchanged [hours ⁻¹].
η_{bat}	Roundtrip charge-discharge battery efficiency [%].
DOD_M	Maximum acceptable battery depth of discharge.
N_T	Number of daily trips made by the train.
PA_r	Power consumed by the ancillary services on-board in section r [kWh].
a, b, c	Constant, linear and quadratic components of the air resistance model [kN, kN/(m/s), kN/(m/s ²)].
g	Gravitational force acceleration: 9.81 m/s ² .
M	Total mass of the train [kg].
η_t	Traction effort efficiency [%].
$\underline{T}, \bar{T}$	Lower and upper limit for the traction effort that the train can exert [kN].
$\underline{a}, \bar{a}$	Lower and upper limit for the acceleration of the train; the lower limit will always be a negative number [m/s ²].
μ_a, μ_b, μ_c	Constant parameters to implement a quadratic model for the upper limit of the traction effort [kN].
s_r	Parameter that is equal to 1 if there is a station at r .
ρ_r	Curvature radius of the track at section r [m].
k_r	Parameter needed to model the reactive force associated with curves [kN · m/Kg].
α_r	Slope of the track at section r [rad].
$\overline{v_r}$	Operational speed limit at section r [m/s].
PT_r^+, PT_r^-	Maximum power consumed/generated by the train at section r due to voltage drop constraints [kW].
λ^B, λ^S	Price at which energy is bought/sold by the train from/to the utility [€/MWh].
β	Parameter indicating the fraction of the energy regenerated that can be used by a different train in the same line.

C. Variables

C_Bat	Energy capacity of the onboard battery [kWh].
t_r	Time at which the train begins to run through section r of the track [s]. Note that $t_1 = 0$ and

the time needed to arrive at the end of the track is t_F .

d_r	Distance between the origin of the track and the position where section r begins [m]. Note that $d_1 = 0$.
v_r	Speed of the train at the beginning of section r [m/s].
a_r	Acceleration of the train during section r [m/s ²].
PB_r/PS_r	Power bought from/sold to the external network in section r [kW].
TEB/TES	Total energy bought from/sold to the external network over the simulation [kWh].
PBD_r/PBC_r	Power discharged/charged from the battery in section r [kWh].
PT_r^-	Train's regenerative braking power in section r [kWh].
PT_r^+	Power consumed by the train in section r [kWh].
RA_r	Resistance to the advance of the train in section r [kN].
T_r	Traction effort in section r of the track [kN].

III. RAILWAYS MODEL

A. SCOPE

In this section we present a mathematical model for the movement of trains that includes a detailed formulation of the dynamics, energy balance, energy exchanges, use of onboard energy storage, and many technical specifications imposed by the infrastructure operator. From a mathematical point of view, we obtain an optimization problem with non-linear constraints that can be solved with the aid of readily available software. For a fixed level of modeling detail, the problem size grows linearly with track length; nevertheless, the presence of nonlinear constraints may lead to non-linear growth in computational effort.

B. OBJECTIVE FUNCTIONS

The optimization problem implementing the operation of a train within a track can be faced from several perspectives. For our model we propose two main objective functions, but many others could also be considered. Firstly, we implement a time minimization objective function, i.e., we try to minimize the amount of time needed to reach the end of the line while complying with all the technical constraints, but without regard to costs. Equation (1) represents this objective function.

$$\text{minimize } t_F \quad (1)$$

Alternatively, we can implement a cost minimization objective function, i.e., we try to minimize the total costs incurred in order to bring the train to the end of the track, complying with all the technical constraints, but without regard to the amount of time needed for the operation. Equation (2) represents this

objective function.

$$\begin{aligned} & \text{minimize} \quad \frac{CBU \cdot C_{Bat}}{B_L \cdot N_T \cdot 365} + \lambda^B \cdot (TEB - \beta \cdot TES) \\ & - \lambda^S \cdot TES \cdot (1 - \beta) \end{aligned} \quad (2)$$

where,

$$TEB = \sum_r (PB_r \cdot (t_{r+1} - t_r)) \quad (3)$$

$$TES = \sum_r (PS_r \cdot (t_{r+1} - t_r)) \quad (4)$$

In (2) the total cost comprises the amortization cost of the battery onboard the train, the cost of acquiring energy and the income obtained from selling energy; note that the option to sell energy back to the network may not be allowed. Additionally, depending on the specific conditions, the energy sold by the train may be coming from regenerative braking or from the battery. Also note that parameter β measures which fraction of the energy generated by regenerative braking can be used by a different train running on the same track; this value must be estimated in advance (see Appendix I). For a value of $\beta = 0$, no energy can be used by other trains and hence, all excess energy has to be sold at network selling price λ^S and all energy must be bought at network energy buying price λ^B ; for values of $\beta > 0$, some of the energy generated by regenerative braking can be used by other trains and hence, some energy does not need to be bought from the external network, saving some money.

Some alternative objective functions can be implemented with small modifications to our model, for example, a fixed time can be assigned to each stop at a station and the model can try to minimize the cost of the operation; or one can fix only the total time and allow the optimization problem to set the intermediate stops at the most convenient times. Moreover, fixing the total time the model could be tweaked in order to obtain the most comfortable operation, minimizing accelerations.

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C. CONSTRAINTS

A number of constraints need to be imposed to model the behavior of the train on the track in order to adequately model the real conditions of operation and the dynamics of such a complex system.

1) TRAIN MOVEMENT AND DYNAMICS

To model the dynamic behavior of the train, the relation between position, speed, acceleration and time must be stated. We assume a model with constant acceleration in each section, hence, within a section, speed can be either constant (no acceleration) or can vary linearly:

$$d_r = d_{r-1} + v_{r-1} \cdot (t_r - t_{r-1}) + \frac{1}{2} \cdot a_{r-1} \cdot (t_r - t_{r-1})^2 \quad \forall r \quad (5)$$

$$v_r = v_{r-1} + a_{r-1} \cdot (t_r - t_{r-1}) \quad \forall r \quad (6)$$

Also, a constraint must be imposed to reflect the fact that the train must stop at the designated stations:

$$v_r = 0 \quad \text{if} \quad s_r = 1 \quad \forall r \quad (7)$$

Note that this constraint implies that the speed is zero at that specific point; this in turn implies that the acceleration in the previous section must be negative and positive in the next.

Due to technical and operative constraints, the speed of the train is limited at each section by a different upper bound related to the type of area that the train is going through; for example, the speed limit is lower when the train is entering a station, going through a pedestrian area or crossing a road. Also, the speed may be limited when the slope of the track exceeds a certain limit. These constraints can be summarized as:

$$v_r \leq \overline{v}_r \quad \forall r \quad (8)$$

Additionally, for reasons of comfort for the passengers, both positive and negative accelerations have to be limited:

$$\underline{a} \leq a_r \leq \bar{a} \quad \forall r \quad (9)$$

2) ENERGY BALANCE

An energy balance equation needs to be imposed in order to be able to account for all the energy exchanges when operating the train (see Notation section).

$$PB_r + PBD_r + PT_r^- = PS_r + PBC_r + PT_r^+ + PA_r \quad \forall r \quad (10)$$

Equation (10) states that the amount of power that enters the main inverter inside the train (bought from the network, discharged from the battery and regenerated from braking) must equal the total amount of power going out of it (sold to the network, used to charge the battery, powering the train wheels and as auxiliary services). All the terms in (10) above must be positive values; also, all of them have upper bounds due to technical reasons.

3) BATTERY STATE OF CHARGE

A constraint must be imposed to model the evolution of the battery's state of charge.

$$SOC_r = SOC_{r-1} + PBC_r \cdot (t_r - t_{r-1}) \cdot \eta_{bat} - PBD_r \cdot (t_r - t_{r-1}) \quad \forall r \quad (11)$$

In (11) above, note that the roundtrip efficiency is only applied to the charged energy. The limits of the state of charge of the battery must also be imposed.

$$C_{Bat} \cdot DOD_M \leq SOC_r \leq C_{Bat} \quad \forall r \quad (12)$$

Note that in our model C_{Bat} is a variable, and hence, the optimal sizing of the ESS is obtained as an added result. Furthermore, there is a limit to the power that can be exchanged by the battery; this limit is computed considering the maximum allowed C_{rate} (CRM) value of operation for the battery.

$$-C_{Bat} \cdot CRM \leq PBC_r - PBD_r \leq C_{Bat} \cdot CRM \quad \forall r \quad (13)$$

The amount of energy stored upon arrival at the end of the track must be at least equal to the energy at the beginning, otherwise, the battery's initial energy will not be accounted for:

$$SOC_1 \leq SOC_F \quad (14)$$

4) TRACTION MODEL

In order to be able to move a train, some reactive forces have to be overcome, these are grouped under variable RA_r , which includes all the forces that oppose the advance of the train in a particular location. There are many different models to estimate the value of RA_r and the selection of the model depends on the technology, the train speed and some other factors. For our model we have chosen a general model that computes RA_r including three different components, the first term is quadratic with speed and is related to air resistance, the second is related to the change in potential energy and the last one is due to track curves. Under these assumptions we arrive at the following expression:

$$RA_r = a + b \cdot v_r + c \cdot v_r^2 + g \cdot M \cdot \sin(\alpha_r) + \frac{M \cdot k_r}{\rho_r} \quad \forall r \quad (15)$$

In extreme cases, like steep downhill slopes, this number can be negative, but in general it will be positive.

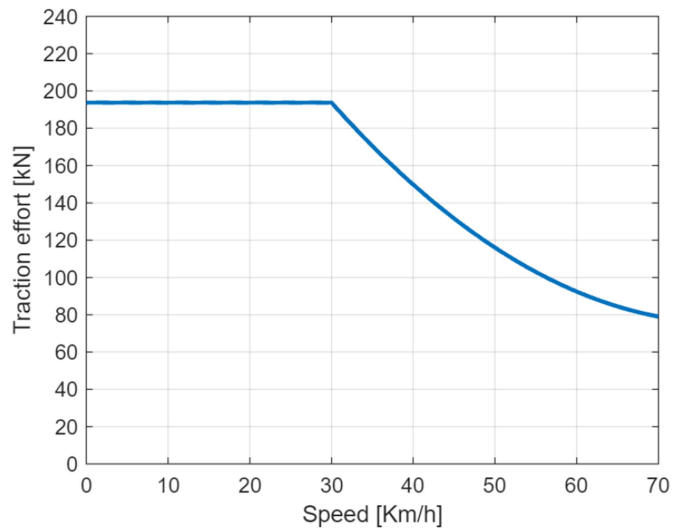


FIGURE 2. Traction effort limits as a function of speed [kN].

In order for the train to move, it must overcome the total resistance to advancement and it must also use force to accelerate all of the mass of the train, this is expressed in the following equation, where both terms are added in order to compute the amount of traction, T_r , that the train needs to exert:

$$T_r = (M \cdot a_r + RA_r) / \eta_t \quad \forall r \quad (16)$$

The value of T_r can be either positive or negative. The electric power needed to obtain a certain amount of traction (PT_r^+) or the amount of energy regenerated (PT_r^-) from a negative value of traction is proportional to the speed of the train; in our model, given that speed is not constant over the sections, we use the average speed as follows:

$$PT_r^+ - PT_r^- = T_r \cdot (v_r + v_{r+1}) / 2 \quad \forall r \quad (17)$$

In addition to the above equations, technological limits must be imposed on the amount of traction that a train can generate, both positive and negative. There are many possible models to implement these technical constraints, we have chosen a model implemented through the following equations:

$$\underline{T} \leq T_r \leq \bar{T} \quad \forall r \quad (18)$$

$$T_r \leq \mu_a \cdot v_r^2 + \mu_b \cdot v_r + \mu_c \quad \forall r \quad (19)$$

Note that both inequalities in (18) impose absolute limits, whereas (19) imposes a different, quadratic limit that depends on the train speed; in our model μ_c is a relatively large number, μ_b is negative and μ_a is small; hence, at slow speeds the limits will be imposed only by (16) whereas at higher speeds, (17) will become limiting. Fig. 2 illustrates this effect.

5) ELECTRIC POWER SUPPLY MODEL

In addition to the previous constraints, our model also includes constraints imposed to the amount of power that the

train can exchange with the power transformers situated along the track to feed the train, due to voltage drop. In particular, the limits imposed by these constraints have to be computed in advance, taking into consideration the section of the electric cables used to feed the trains, their specific composition, the current limits of the cables, the locations of all feeding substations and the distance from a specific section to the nearest substation. Once these computations are performed, the following new constraints are imposed:

$$PT_r^+ \leq \overline{PT_r^+} \quad \forall r \quad (20)$$

$$PT_r^- \leq \overline{PT_r^-} \quad \forall r \quad (21)$$

IV. CASE STUDY

In this section we present a real case study that was analyzed using our model; solutions are obtained and conclusions are drawn. Implementation details are provided.

A. TRAIN AND TRACK

The simulated train and track in this work is the green line under construction in Jerusalem, Israel. This is a new 16.5 km project, that was approved by the state of Israel as a PPP project (Public-Private Partnership), including Spanish company CAF and Israeli company SHAPIR. This line is part of a larger proposed system: the Jerusalem LRT Network (J-Net), comprising the already functional red line and some other proposed ones. The green line connects Mount Scopus and French Hill in the north of Jerusalem with the neighborhood of Gilo in the south; in its middle section, the line meets the red line and goes near the Old City of Jerusalem. The maximum slope of the line is 9.0 %. The line features 33 stations and is fed in DC at a rated 750 V through 13 transformer-rectifying substations.

The DC supply infrastructure along the Green Line includes 13 reversible transformer substations (RTS). This configuration allows the energy recovered through regenerative braking to be directly reused by other trains operating along the line, regardless of their precise location or direction. Such capability maximizes the efficiency of the system by enabling dynamic energy sharing between trains, enhancing both sustainability and operational cost-effectiveness.

The trains selected for this project are manufactured by CAF and feature 5 articulated car bodies. The maximum capacity is 218 passengers. The total weight of the train is 124 Tn, its maximum acceleration and deceleration are 0.9 and 0.8 m/s², respectively; its maximum rated speed is 70 km/h, and its maximum traction effort is 194 kN.

Based on data obtained from the real plans of the project, a very detailed set of data defining the track has been obtained as the first step of the work needed to perform the simulations shown in this work. The result is an extensive data set comprising 2154 rows, one for each section of the track. For each section, the data set contains the following information: i) length of the section; ii) slope of the section; iii) horizontal curvature radius of the section; iv) vertical curvature radius

of the section; v) interaction with other infrastructures, e.g., crossing a road, elevated pass, pedestrian area, etc.; vi) maximum allowed speed; vii) distance to nearest substation. Also, all stations are marked on the data set. Note that all of these parameters must be constant within a section along the track, and therefore, whenever one of these values changes, a new section must be defined; this explains why we observe such a high number of sections for a track of relatively short length. To illustrate the level of detail in our data, note that the average length of the sections is 14.98 m, with 140 being shorter than 1 m. The data set has been extended to cover both directions of the Green Line, from Gilo to Mount Scopus and back, allowing for full bidirectional simulation and analysis. Further efforts were required to obtain the values of a large number of additional parameters that our formulation needs in order to precisely model the train movement. These parameters include data regarding the rolling stock, the electric facilities feeding the track, etc. To the best of our knowledge, no prior work in the literature presents a simulation of the track and the energy exchanges of a light train with comparable level of detail.

B. SIMULATIONS AND RESULTS

A total of 16 simulations were conducted for this study, each combining different settings for the objective function (minimizing cost or time), the presence of an onboard energy storage system (ESS), the use of regenerative braking (R.B.), and the maximum allowed speed (50 or 70 km/h). The goal was to explore the effects of these variables on energy consumption and operational performance of the light rail along the entire Green Line route.

All simulations were implemented under the GAMS programming environment using the CONOPT solver. The computational setup included a standard computer equipped with an Intel i5 processor (3.0 GHz) and 8 GB of RAM. The computation times varied depending on the scenario complexity.

The analysis presented in this paper focuses on the round-trip route, where the train departs from Gilo, travels to Mount Scopus, and returns to Gilo. This configuration reflects real operational conditions and enables full-cycle energy assessment. Table 1 summarizes the results of all round-trip simulations, including total cost, trip duration and battery size for each case. Each row corresponds to a specific operational configuration, combining different values for the objective function, presence of onboard energy storage system (ESS), regenerative braking (R.B.) and maximum speed. For each case, the table reports the total cost in euros, total round-trip duration in minutes, and the optimal energy storage capacity if applicable.

This structured layout enables a clear comparison of trade-offs between energy consumption, operational efficiency, and cost optimization under multiple realistic scenarios. The simulations were designed to replicate full daily round-trip operations between Gilo and Mount Scopus stations and back, using realistic performance and infrastructure constraints.

TABLE 1. Simulation Cases and Results (Round-Trip)

#	Ob	ESS	R. B.	Max. speed [Km/h]	Total cost [€]	Time [min]	ESS Capacity [kWh]
1	C	No	No	50	62.3886	131.34	0.0
2	C	Yes	No	50	62.3886	131.34	0.0
3	C	No	Yes	50	35.1119	119.80	0.0
4	C	Yes	Yes	50	34.4733	119.21	186.62
5	C	No	No	70	62.3854	131.32	0.0
6	C	Yes	No	70	62.3854	131.32	0.0
7	C	No	Yes	70	35.1080	119.75	0.0
8	C	Yes	Yes	70	34.4695	119.18	188.06
9	T	No	No	50	78.2799	116.65	0.0
10	T	Yes	No	50	78.2799	116.65	0.0
11	T	No	Yes	50	36.2003	116.65	0.0
12	T	Yes	Yes	50	36.2003	116.65	0.0
13	T	No	No	70	79.2525	116.43	0.0
14	T	Yes	No	70	79.2525	116.43	0.0
15	T	No	Yes	70	36.3327	116.43	0.0
16	T	Yes	Yes	70	36.3327	116.43	0.0

In Table 1, the first column identifies the simulation case number, the second column indicates the objective function applied in the simulation (Cost minimization or Time minimization), the next two columns specify whether the scenario includes an onboard Energy Storage System (ESS) and whether regenerative braking (R.B.) is enabled, the fifth column indicates the highest train velocity allowed in the scenario, the sixth column shows the total cost for completing a full round trip, based on electricity purchases and the optimized usage of the ESS (remember that whenever a battery is included, its cost is included in the objective function); the seventh column presents the total travel time, in minutes, required to complete the round trip under each specific configuration, including time at the stops, and finally, the last column shows the optimized battery capacity value.

From Table 1, it is clear that when the objective function is time minimization, the results are consistent across configurations regarding regenerative braking or ESS availability. For example, cases 9, 10, 11 and 12 all present the same travel time (116.65 min), although they differ in operational cost. In the remaining cases with time minimization (cases 13–16) note that the travel time is also the same (116.43 min), differing by just saving 13 seconds thanks to the increased maximum speed of 70 km/h.

In contrast, when cost minimization is used as the objective (cases 1–8), travel times are increased between 3 and 15 minutes and costs are reduced by different amounts, depending on the configuration; note also that the inclusion of regenerative braking yields a significant cost reduction (compare cases 1 vs. 3 or 2 vs. 4). Moreover, if regenerative braking is allowed, the use of an onboard energy storage system (ESS) further decreases the operational cost (e.g., case 3 vs. case 4). However, in scenarios where regenerative braking is not available, the addition of an ESS does not provide any advantage (see case 1 vs. case 2 or case 9 vs. case 10).

Table 2 shows, for each of the 8 possible technical configurations, a comparison of the results obtained for the two different objective functions, highlighting the cost and time

TABLE 2. Time Minimization vs. Cost Minimization

Cases	ESS	R. B.	Max. speed [Km/h]	Cost diff. [€]	Cost diff. [%]	Time diff. [min]	Time diff. [%]
1 vs. 9	No	No	50	+15.891	+25.47%	-14.69	-11.18%
2 vs. 10	Yes	No	50	+15.891	+25.47%	-14.69	-11.18%
3 vs. 11	No	Yes	50	+1.088	+3.01%	-3.15	-2.63%
4 vs. 12	Yes	Yes	50	+1.727	+5.00%	-2.56	-2.15%
5 vs. 13	No	No	70	+16.867	+27.04%	-14.89	-11.34%
6 vs. 14	Yes	No	70	+16.867	+27.04%	-14.89	-11.34%
7 vs. 15	No	Yes	70	+1.225	+3.48%	-3.32	-2.77%
8 vs. 16	Yes	Yes	70	+1.863	+5.40%	-2.75	-2.31%

difference between cases. For example, from the first row of Table 2 it is clear that when neither ESS nor RB is available and maximum speed is 50 km/h, then time minimization causes a cost increase of 15.891€ (a 25.5 % cost increase), while shortening the travel time by 14.69 minutes (an 11.18% time decrease); note that this pattern is repeated for all cases, when switching to time minimization, the percentage increase in cost is always greater than the percentage decrease in time. Hence, it is important to highlight that time minimization always results in a higher cost compared to cost minimization; and that the difference in travel time is usually relatively small. Therefore, a cost-based optimization might provide a more efficient trade-off between energy usage and performance. Also note that, for configurations that allow for regenerative braking, the differences are small among the two possible objective functions.

C. DETAILED ANALYSIS OF CASE 4

In the following analysis we focus on case 4, which was selected as the reference case for its representative and energy-efficient behavior. In this scenario, the train operates under cost minimization with both regenerative braking and ESS enabled; the maximum allowed speed is 50 km/h (13.89 m/s) and the value of the β parameter is 0.3. The total trip duration is 119.2 minutes, just 2.5 minutes longer than the time-minimized case and the cost is the smallest of all studied cases, at just €34.47. The optimized battery capacity for this case is 186.62 kWh. Fig. 3 presents the train's velocity profile over the full round trip, where frequent drops to zero correspond to station stops and terminal reversals, also note the speed limit. Fig. 4 presents a detailed version of Fig. 3 between 10000 m and 14000 m.

The battery's energy evolution throughout the route is shown in Fig. 5. Notably, the battery reaches its minimum state-of-charge at approximately 11000 m along the route and again around 22000 m, which correspond to the high points of elevation. Also, around 5000 m, the battery is almost full, as this point is at a very low elevation point and a lot of regenerative braking takes place between 4000 m and 5000 m.

In Figs. 6 and 7 the amount of energy charged and discharged at each point over the track is presented. Note that these graphs reveal the ESS's active role in smoothing power fluctuations along the trip. The net battery energy profile displays a cyclic pattern, with the state-of-energy oscillating

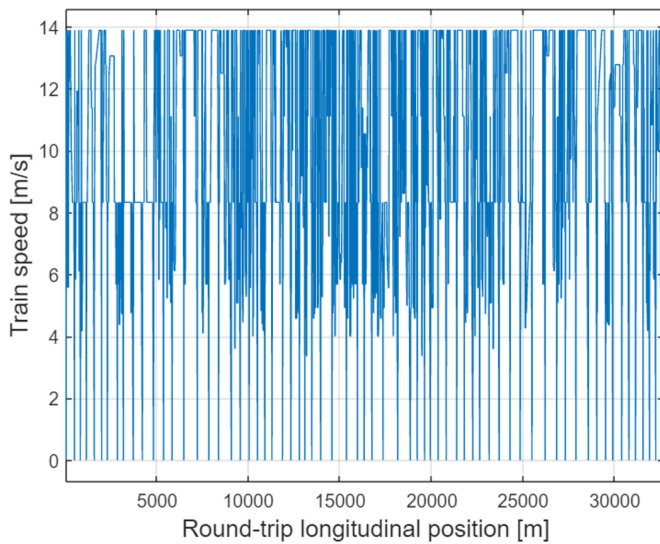


FIGURE 3. Train speed round trip [m/s].

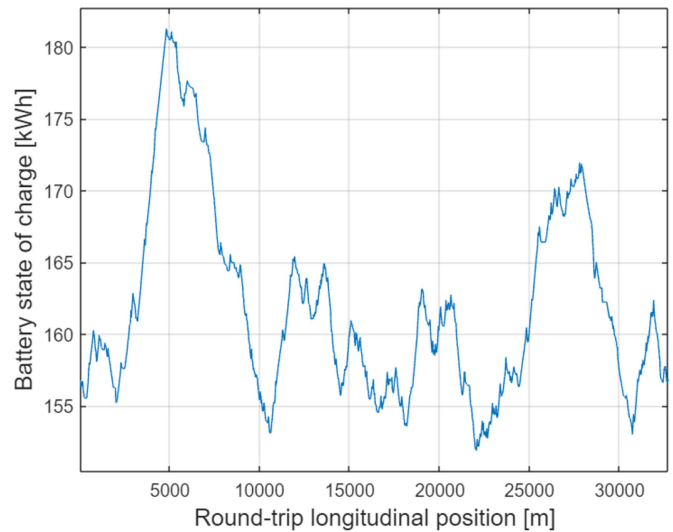


FIGURE 5. Train battery state of charge [kWh].

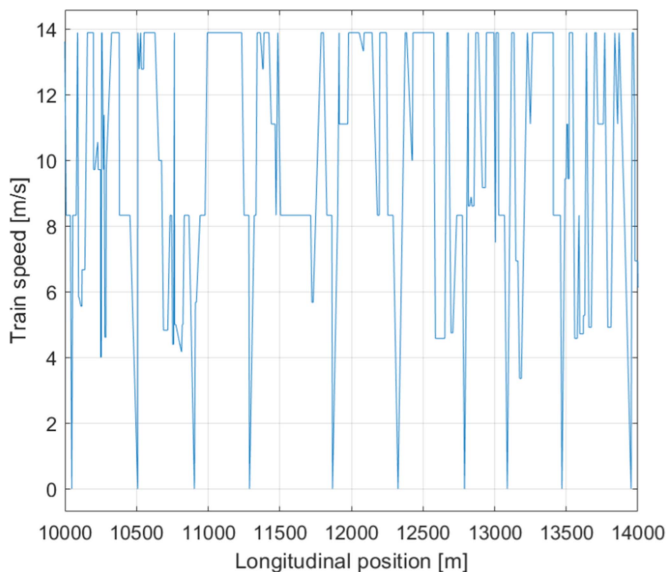


FIGURE 4. Detail of train speed between 10000 m and 14000 m.

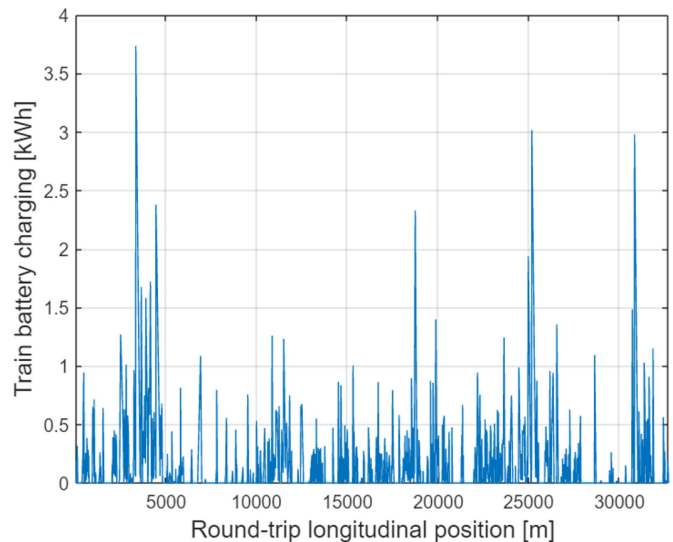


FIGURE 6. Battery charging energy [kWh].

between 150 kWh and 185 kWh, in line with the driving dynamics and energy exchange requirements.

It is important to emphasize that the battery is not operated in a way that gets it fully charged and discharged between specific points. This may seem counterintuitive and might suggest a smaller battery would suffice; however, in practice, battery sizing is often constrained not only by energy capacity but also by power requirements. These are interconnected through the battery's C-rate limitations, making the selected capacity optimal from both energy and power perspectives.

Fig. 8 shows the traction force applied by the train, with peak values observed during uphill climbs or when departing from stations. Note that there are positive and negative values of traction. Fig. 9 presents a detailed version of Fig. 8 between

10000 m and 14000 m. In Fig. 10, the energy consumption profile is shown, indicating the power drawn either from the grid or from the onboard energy storage system (ESS), while Fig. 11 highlights the energy returned to the system through regenerative braking, most prominent during downhill segments or scheduled stops.

Fig. 12 presents the energy sold back to the grid along the round trip; this occurs sporadically and remains modest, with only a few instances exceeding 2 kWh, primarily at the start and end of the route. Fig. 13 presents the energy purchased from the grid.

Unlike previous figures, Figs. 14 and 15 show power data instead of energy data. Fig. 14 shows both the train's power demand and the maximum amount of power that can be

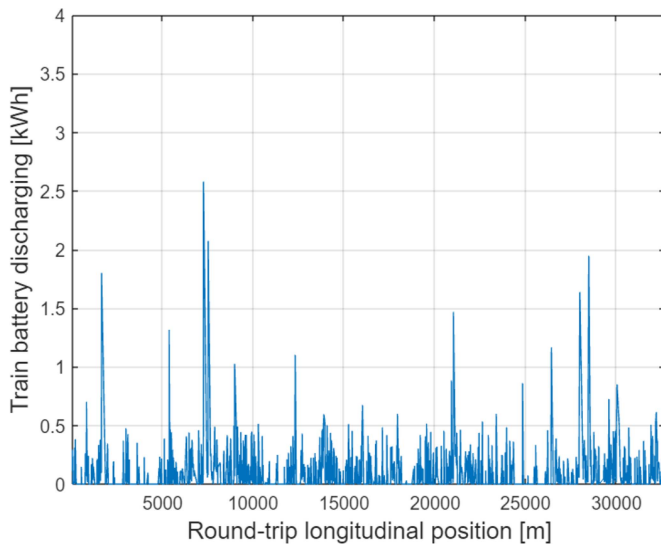


FIGURE 7. Battery discharging energy [kWh].

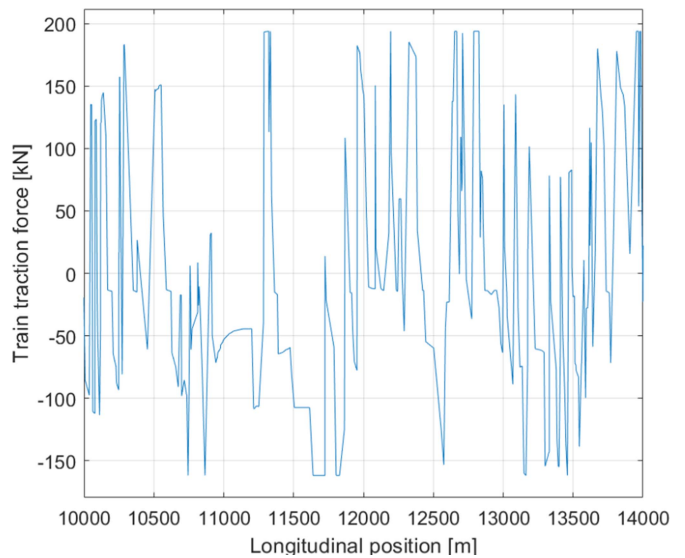


FIGURE 9. Detail of train traction force between 10000 m and 14000 m.

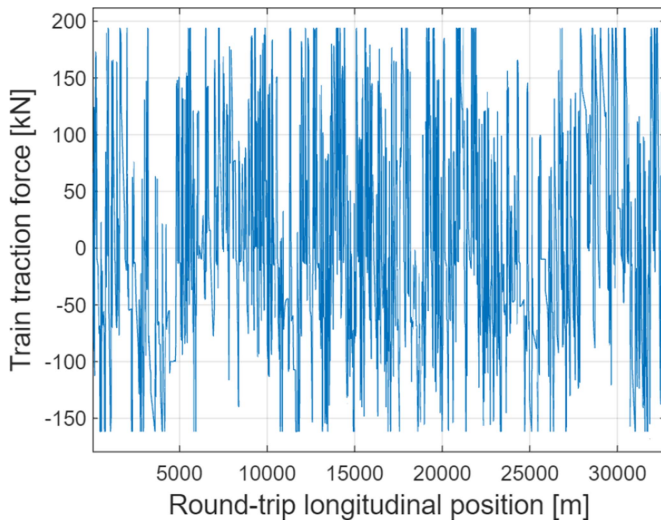


FIGURE 8. Train traction force round trip [kN].

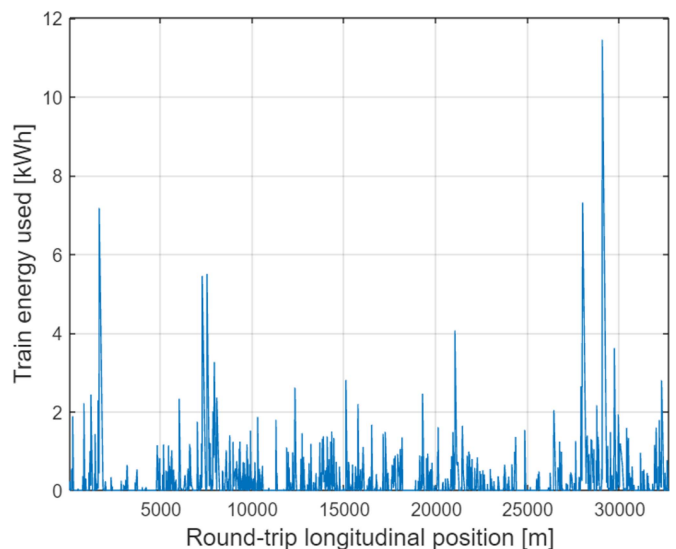


FIGURE 10. Train energy demanded through regenerative braking round trip [kWh].

consumed along the line; since voltage drop is explicitly considered in the simulations, this maximum value varies with position. Power demanded peaks sharply during acceleration phases, occasionally surpassing 1.8 MW, especially near locations such as 20000 m. Also, the amount of power supplied by the grid is limited in some segments due to voltage drop, as observed in several locations; for example, at around 16000 m, where the demand is capped at approximately 1 MW. Additionally, Fig. 15 shows power obtained from the train, specifically regenerative braking; this procedure enables significant power recovery, with instantaneous values frequently exceeding 1.2 MW and reaching over 1.4 MW near 25000 m.

To validate the results of the proposed model, the system-level simulation outputs, specifically train speed profiles, traction energy consumption, and the energy savings achieved

through regenerative braking and train-to-train energy exchange, were qualitatively compared with simulations developed within the CAF-Shapir J-NET Green Line project. These design-stage simulations are based on commercial operating assumptions and capture the characteristic energy consumption patterns during acceleration, cruising, and braking, as well as the recovery of energy through regenerative braking and its utilization by other trains operating in acceleration phases. Although these studies are proprietary and cannot be reproduced in this paper, a strong qualitative consistency was observed in the speed-profile structure, energy consumption behavior, and the magnitude of energy savings enabled by regenerative braking and inter-train energy exchange. This consistency between the two sets of simulations

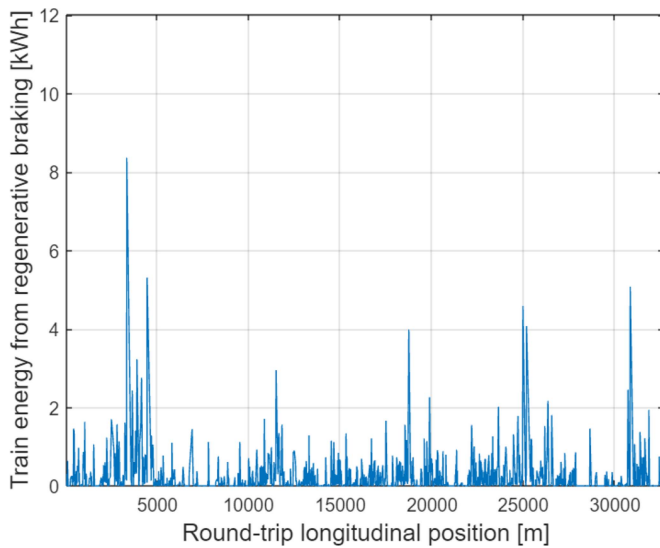


FIGURE 11. Train energy generated through regenerative braking round trip [kWh].

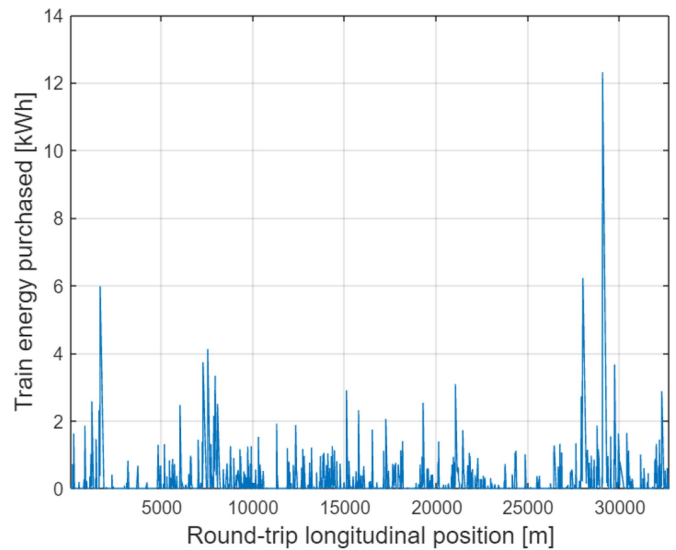


FIGURE 13. Train energy bought round trip [kWh].

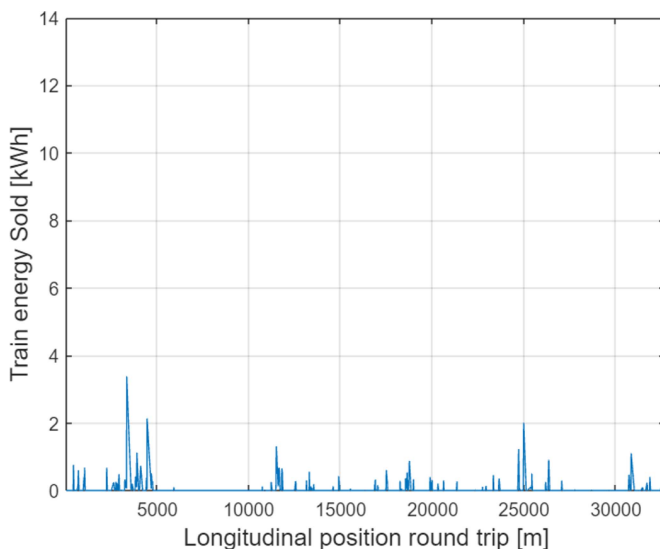


FIGURE 12. Train energy sold round trip [kWh].

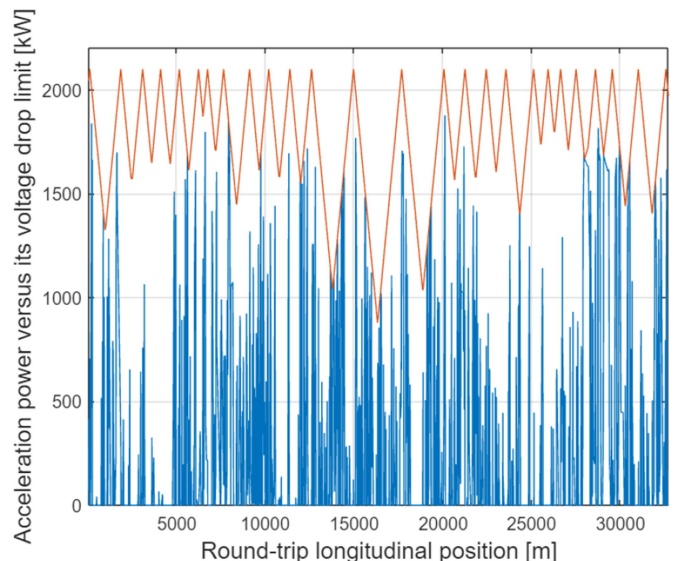


FIGURE 14. Power used by the train [kW].

supports the validity of the proposed model and its relevance for the assessment of energy efficiency improvements in real-world railway operations.

D. SENSITIVITY ANALYSIS FOR PARAMETER β

The real time feasibility of all energy transfers among trains can be relatively uncertain as it depends not only on the train adhering to schedule, but also on the rest of the trains on the line also strictly moving on time along the line; this uncertainty will have an effect on the objective function and in fact on the ability to exchange energy; in order to replicate this effect, in this section a sensitivity analysis is performed regarding the β parameter; note that lower values of β indicate fewer options to exchange energy. A set of simulations similar

to case 4 discussed above are run for different values of β between 0 and 0.5 at 0.05 intervals. The results in terms of the objective function and time duration are presented in Fig. 16. Note that the influence of the β parameter on travel times is rather small, approximately 1 minute difference when β changes from 0.0 to 0.5; in terms of cost, the effect is also small, with 0.5 € difference between the two extreme cases presented in Fig. 16.

E. MODES OF OPERATION AND TIMETABLE

Beyond the analysis of energy optimization by a train for a single round trip, train operators are also interested in studying the cost and amount of energy needed in order to provide

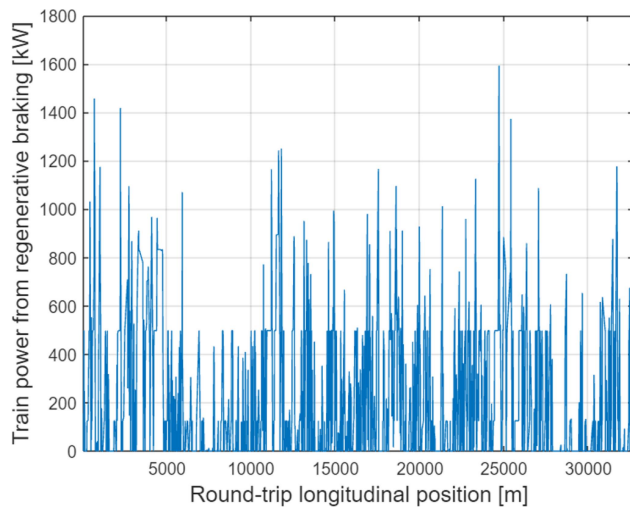


FIGURE 15. Power regenerated by the train [kW].

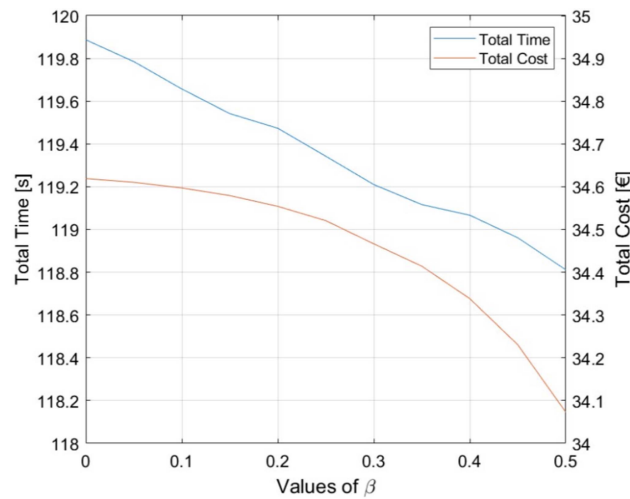


FIGURE 16. Sensitivity analysis for parameter β .

specific levels of service to customers; to this end, three different modes of operation have been proposed for the Green Line, based on realistic passenger demand patterns. In all considered modes of operation, service is divided into peak and off-peak intervals, for peak hours one train departs every 5 minutes from each terminal and on off-peak hours the frequency is reduced to one train every 15 minutes. In *Operation Mode 1*, peak hours are from 5:00 to 10:00 and from 16:00 to 22:00 and off-peak hours are from 10:00 to 16:00 and from 22:00 to 1:00. *Operation Mode 2* is similar but the interval from 10:00 to 16:00 is also part of the peak-hours. Finally, *Operation Mode 3* considers all hours of operation as peak hours and hence assumes a uniform high-frequency of trains departing every 5 minutes from both terminal stations. *Operation Mode 3* seems to be the most probable to be implemented and hence is the main focus of study in this section.

In order to meet this 5-minute frequency target, it is clear that at least 24 trains are needed (note that each train takes approximately 2 hours to complete a round trip); also note

TABLE 3. Arrival Timetable for the First Train of the Day

#	Station (direction Mount-Scopus)	Arrival time	Station (direction Gilo)	Arrival time
1	Gilo	05:00	Mount-Scopus	05:00
2	Ha-Margalit	05:03	Shocken	05:03
3	Yaar Gilo	05:04	French Hill	05:05
4	Dagan	05:06	Clermont Ganneau	05:08
5	Ha Khasida	05:07	Katzir	05:10
6	Ha Snunit	05:09	Mahal	05:12
7	Leshem	05:10	Karl Neter	05:13
8	Ha Ganenet	05:12	Eretz Khafetz	05:15
9	Ha Rosmarin	05:13	Shmuel Ha-Navi	05:16
10	Shabtai Ha-Negbi	05:15	Eli Ha Cohen	05:18
11	Begin South	05:17	Yirmiyahu	05:20
12	Ikhud Ha-kfar	05:19	Romema	05:22
13	Nakhel Refaim	05:21	Binyanei Ha-Uma	05:24
14	Yehuda Ha-Nasi	05:23	Govat - center	05:26
15	Shakhal	05:24	Givat-Ram	05:28
16	Bayt	05:27	Balfour	05:30
17	Mini Campus	05:29	Mini-Campus	05:32
18	Balfour	05:31	Bayt	05:34
19	Givat-Ram	05:32	Shakhal	05:36
20	Govat-Center	05:35	Yehuda Ha-Nasi	05:38
21	Binyanei Ha-Uma	05:36	Nakhel Refaim	05:40
22	Romema	05:39	Ikhud Ha-Kfar	05:42
23	Yirmiyahu	05:40	Begin South	05:44
24	Eli Ha-Cohen	05:42	Shabtai Ha-Negbi	05:46
25	Shmuel Ha-Navi	05:43	Ha Rosmarin	05:47
26	Eretz Khafetz	05:45	Ha Ganenet	05:49
27	Karl Neter	05:47	Leshem	05:50
28	Mahal	05:48	Ha Snunit	05:52
29	Katzir	05:49	Ha Khasida	05:53
30	Clermont Ganneau	05:52	Dagan	05:55
31	French Hill	05:54	Yaar Gilo	05:56
32	Shocken	05:57	Ha-Margalit	05:58
33	Mount-Scopus	05:59	Gilo	06:00

that each train will have to make around 10 full round trips each day, for a total of 240 round trips per day. With these data in mind Table 3 can be obtained, presenting the arrival Timetable for the first train of the day for each station; these are trains that depart at 5:00 from both Gilo and Mount Scopus terminals. The table lists all 33 intermediate stations in order, along with the calculated arrival time at each station in both directions. This schedule is derived from the simulation results and reflects realistic trip durations under standard operating conditions. Such information is essential for operational planning, passenger communication, and alignment with service frequency targets. The data also enables validation of time-optimized simulation scenarios against real-world expectations.

To evaluate the daily energy efficiency under realistic operating conditions, this study emphasizes the results obtained in scenario 4. Under this setup, and considering a total of 240 daily round-trips, the total daily cost is €8273.52. Under these circumstances, the system managed to save an amount of €270.14 by sending energy to other trains instead of selling back to the grid, highlighting the benefits of being able to redistribute energy during train operation. This behavior clearly improves energy efficiency.

The results confirm that this configuration strikes an effective balance between cost savings, energy recovery, and operational realism. The ESS plays a complementary role

to regenerative braking, ensuring that excess energy is not wasted even when the network's ability to absorb it externally is limited. From a planning perspective, this suggests that investments in onboard ESS are economically justified only when paired with regenerative capabilities and optimized control logic.

V. CONCLUSION

This study developed and validated a comprehensive optimization framework for urban railway operation, capable of integrating dynamic train behavior, energy exchanges, and realistic infrastructure constraints into a unified mathematical model. The formulation incorporates detailed traction dynamics, energy balance, and battery management, leading to a non-linear optimization problem that can be efficiently solved using commercial solvers. The model's flexible structure allows for adaptation to various train technologies, grid configurations, and service patterns, providing a strong foundation for data-driven decision-making in railway planning. Furthermore, its outputs, including speed profiles, battery state evolution, and operational timetables, can provide extremely useful information to LRT operators.

The proposed model was applied to the Jerusalem Light Rail Green Line, using detailed topographic and operational data comprising 2154 track segments. Two main optimization strategies were considered, time minimization and cost minimization, each representing a different operational perspective. Sixteen scenarios were simulated; the results identified scenario 4 (cost minimization with both regenerative braking and ESS) as an adequate configuration. The results demonstrate that cost minimization leads to significantly lower energy costs and to small increases in travel duration, hence, a cost minimization policy provides a more balanced and sustainable trade-off between performance and efficiency.

Our study also confirms the model's practical applicability by extending the analysis to a full-day operation, with 5-minute train intervals during all service hours.

Overall, the proposed model offers a robust and flexible tool for optimizing urban rail systems, enabling operators to increase energy efficiency, reduce operational costs, and support the transition toward more sustainable public transportation networks.

APPENDIX

In this study, parameter β is defined as the fraction of regenerative braking energy that the traction power network can effectively transfer, in real time, to other trains operating in the same line, relative to the total regenerated energy. The estimation of β was carried out in two main stages:

STAGE 1: Baseline simulation. In the first stage, the 16 optimization models were executed with β set to zero, implying that no energy transfer between trains was assumed. This baseline simulation provides the values of traction power, regenerative braking power, train speeds, and travel times.

STAGE 2: β estimation. For each of the 16 scenarios, in this second stage, the power profiles (traction and braking) of

all the trains running simultaneously on the track are generated, assuming that they are identical but with a fixed delay equal to five minutes. Time intervals are then identified in which the braking of the train under consideration overlaps with the acceleration of another train or conversely, the acceleration of the studied train coincides with a different train braking. For each of these overlapping periods, it is verified that the energy exchange complies with network constraints such as minimum voltage and maximum current limits. The transferable regenerative energy is computed as the total amount of energy that could be exchanged between trains during these overlapping intervals and the value of β is obtained as the ratio between this transferable energy and the total regenerative energy. This procedure is repeated for each scenario; yielding one β value per case.

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